

Code: 23BS1201

**I B.Tech - II Semester – Supplementary Examinations  
DECEMBER 2025**

**DIFFERENTIAL EQUATIONS & VECTOR CALCULUS  
(Common for ALL BRANCHES)**

Duration: 3 hours

Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.

2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.

3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.

4. All parts of Question paper must be answered in one place.

**PART – A**

|      |  |
|------|--|
| 1.a) | Find the integrating factor that makes the equation $(3xy^2 - y^3)dx - (2x^2y - xy^2)dy = 0$ exact.  |
| 1.b) | Solve $xdy + (y + x)dx = 0$ .  |
| 1.c) | Find the particular integral of $(D^2 + 6D + 9)y = 2e^{-3x}$ .   |
| 1.d) | Find the complementary function of $(D^2 + 4D + 5)y = -2\cosh x$ .   |
| 1.e) | Form the partial differential equation by eliminating the arbitrary functions from $z = f(x + at) + g(x - at)$ .   |
| 1.f) | Solve $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$ .                       |
| 1.g) | Find the greatest value of the directional derivative of the function $f = x^2yz^3$ at $(2, 1, -1)$ .  |
| 1.h) | Find the constants a, b, and c if the vector $\vec{f} = (2x + 3y + az)\mathbf{i} + (bx + 2y + 3z)\mathbf{j} + (2x + cy + 3z)\mathbf{k}$ is Irrotational. |
| 1.i) | Write the statement of Gauss Divergence's theorem.   |
| 1.j) | Given $F(t) = (5t^2 - 3t)\mathbf{i} + 6t^3\mathbf{j} - 7t\mathbf{k}$ , then evaluate $\int_{t=2}^4 F(t) dt$ .  |

## PART – B

|                 |    |   | Max.<br>Marks |
|-----------------|----|---|---------------|
| <b>UNIT-I</b>   |    |   |               |
| 2               | a) | Solve the differential equation $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{1}{(1+x^2)^2}$ given that $y = 0$ when $x = 0$ .   | 5 M           |
|                 | b) | A body originally at $80^{\circ}\text{C}$ cools down $60^{\circ}\text{C}$ in 20 minutes, the temperature of the air being $40^{\circ}\text{C}$ . What will be the temperature of the body after 40 minutes from the original? | 5 M           |
| <b>OR</b>       |    |   |               |
| 3               | a) | Solve $x^2y dx - (x^3 + y^3) dy = 0$ .  | 5 M           |
|                 | b) | A bacterial culture growing exponentially, increases from 200 to 500 grams in the period of 6 am to 9 am. How many will be present at noon.   | 5 M           |
| <b>UNIT-II</b>  |    |   |               |
| 4               | a) | Solve $(D^2 - 4)y = 2\cos^2 x$ .  | 5 M           |
|                 | b) | Solve $(D^2 - 7D + 6)y = e^{2x}(1+x)$ .   | 5 M           |
| <b>OR</b>       |    |   |               |
| 5               |    | Using method of variation of parameters solve $(D^2 + 2D + 1)y = xe^{2x}$ .   | 10 M          |
| <b>UNIT-III</b> |    |   |               |
| 6               | a) | Form the partial differential equation by eliminating the arbitrary function from $z = (x+y)\phi(x^2 - y^2)$ .  | 5 M           |
|                 | b) | Solve $(mz - ny)p + (nx - lz)q = ly - mx$ .   | 5 M           |

| <b>OR</b>      |    |   |      |
|----------------|----|---|------|
| 7              | a) | Solve the PDE $(D^2 - DD')$ $z = \cos x \cos 2y$ .  | 5 M  |
|                | b) | Solve the PDE $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = y \sin x$ . | 5 M  |
| <b>UNIT-IV</b> |    |   |      |
| 8              | a) | Find the directional derivative of a scalar function $\phi = x^2 - 2y^2 + 4z^2$ at $(1, 1, -1)$ in the direction $2i + j - k$ .                   | 5 M  |
|                | b) | Find the angle between the normal's to the surface $\phi = x^2 + y^2 - z$ at the points $(1, 2, 3)$ and $(2, 3, 4)$ .                             | 5 M  |
| <b>OR</b>      |    |   |      |
| 9              | a) | Check whether the vector field $\vec{f} = (2xy + z^3)i + x^2j + 3xz^2k$ , is irrotational or not? If so, find its scalar potential function.      | 5 M  |
|                | b) | If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then show that $\nabla(r^n) = nr^{n-2}\vec{r}$ , where $r =  \vec{r} $ .                            | 5 M  |
| <b>UNIT-V</b>  |    |   |      |
| 10             | a) | Find the work moving the particle in the force field $F = 3x^2i + j + zk$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$ .               | 5 M  |
|                | b) | Apply the Green's theorem to evaluate $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$ where 'C' is the region bounded by $y = x^2$ and $y = \sqrt{x}$ .  | 5 M  |
| <b>OR</b>      |    |   |      |
| 11             |    | Verify Stoke's theorem for a vector field defined by $\vec{f} = -y^3i + x^3j$ , in the region $x^2 + y^2 \leq 1, z = 0$ .                         | 10 M |